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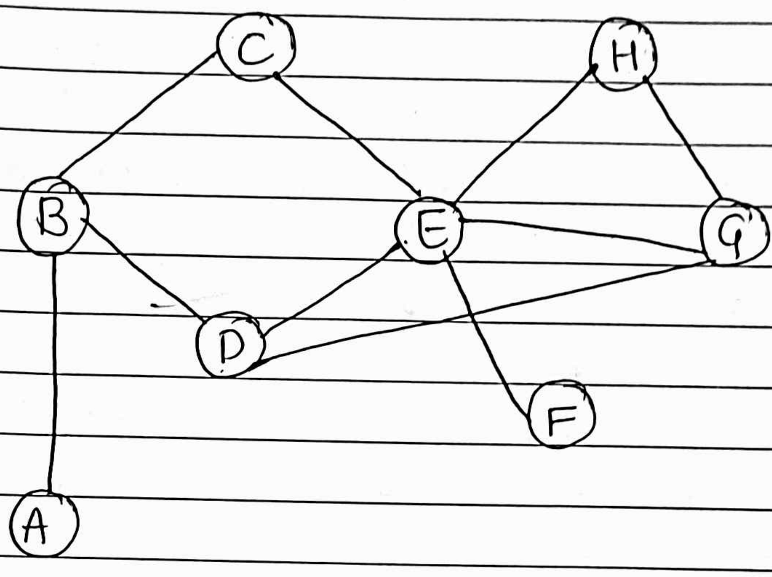
(Religious Jain Minority)

Subject: ~~Internet Programming~~

Q. Consider this graph:



Answer the following questions about this graph.

a) How many nodes are in the network?

Ans There are 8 nodes - named as A, B, C, D, E, F, G, H in the given network.

b) How many edges are in the network?

Ans There are 10 edges in the given network. They are as:

~~AB~~ A⟷B   C⟷E   E⟷H   H⟷G
B⟷D   D⟷E   E⟷G
B⟷C   D⟷G   E⟷F

c) Is this graph directed or undirected?

Ans The given graph is undirected as there is no pointing edges to any node.

d) Create an adjacency list for this graph.

Ans Adjacency list for the given graph is as follow:

1) A, B
2) B, D
3) B, C
4) C, E
5) D, E
6) E, H
7) E, G
8) E, F
9) D, G
10) H, G.

e) Create an adjacency matrix for this graph.

Ans An adjacency matrix for the given graph are as follows:

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$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

f) What is the length of the shortest path from node A to node F?

Ans The shortest path from node A to node F is of length 4 which takes two different path as follows:

1) $A \xrightarrow{1} B \xrightarrow{2} D \xrightarrow{3} E \xrightarrow{4} F$

2) $A \xrightarrow{1} B \xrightarrow{2} C \xrightarrow{3} E \xrightarrow{4} F$

g) What is the largest clique in this network? How many cliques of that size are there?

Ans When all nodes in a group are connected to one another, it is called a clique.

The largest clique in the given network is of size 3

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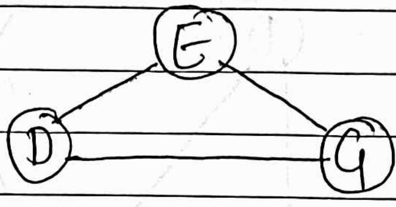
In given network of size 3, there are 2 cliques as follows:

1



2

h) How many connected components are there in this network?

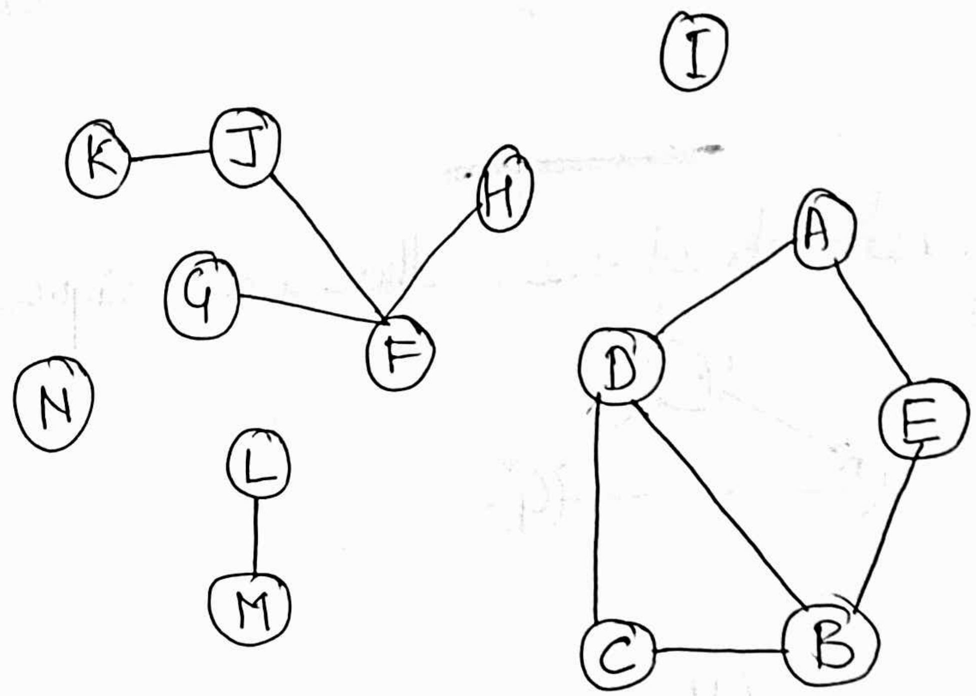
Ans A connected component of an undirected graph is a subgraph in which each pair of nodes is connected with each other via a path.

In connected components, all the nodes are always reachable from each other.

Thus, in the given network there is one connected component, it is as follows:

$C1 = \{A, B, C, D, E, F, G, H\}$

Q.



→ How many connected components are there in this network?

Ans There are 5 connected components in this network. They are as follows:

$C1 = \{K, J, F, G, H\}$

$C2 = \{A, E, B, C, D\}$

$C3 = \{L, M\}$

$C4 = \{N\}$

$C5 = \{I\}$

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i) Draw the 1.5 ego network for node E (without including node E in the graph). How many singletons are in the ego network?

Ans If we want to see only E's neighbours and their connections, it is called a 1.5 degree egocentric network.

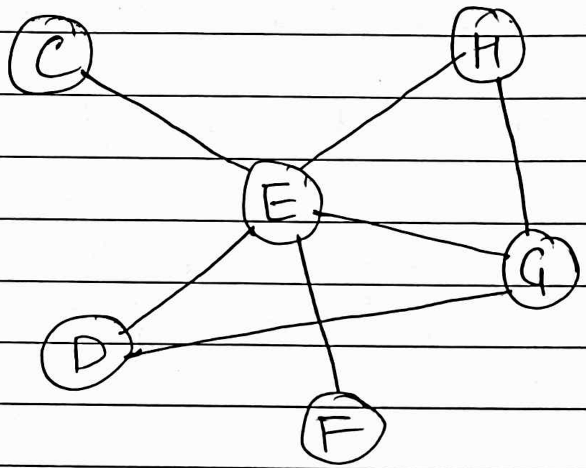

fig: 1.5-degree egocentric network of node E

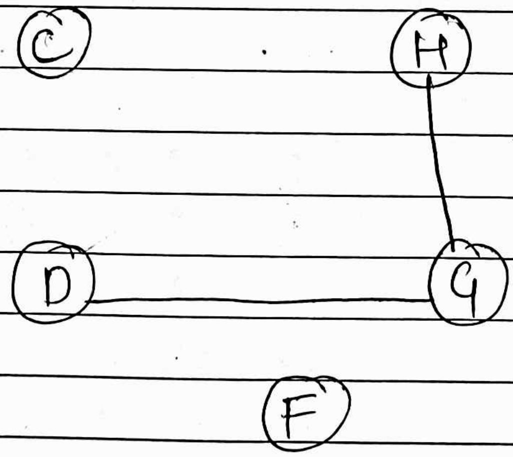

fig: 1.5-egocentric network of node E with E excluded.

Thus, from the above ego network there are 2 singletons i.e C and F

j) Are there any hubs in the network? If so, which nodes and why is it a hub?

Ans Hubs are the most connected nodes in the network.

Yes, there hub in the network. Node E is a hub in the network.
Node E is the most connected nodes with 5 edges to it, which is highest compare to other nodes in the network.

k) What is the degree of each node in the network?

Ans The degree of a node is the number of edges connected to that node.

Degree of node A is 1
Degree of node B is 3
Degree of node C is 2
Degree of node D is 3
Degree of node E is 5
Degree of node F is 1
Degree of node G is 3
Degree of node H is 2

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1) What is the degree distribution for this graph?

Ans To create a degree distribution following steps are to be followed:

Step 1) Calculate the degree for each node in the network.

Node	Degree
A	1
B	3
C	2
D	3
E	5
F	1
G	3
H	2

Step 2) Count how many nodes have each degree. This is totaled for each degree, including those for which there are no nodes with that count.

Degree	No. of Nodes have that Degree
1	2
2	2
3	3
4	0
5	1

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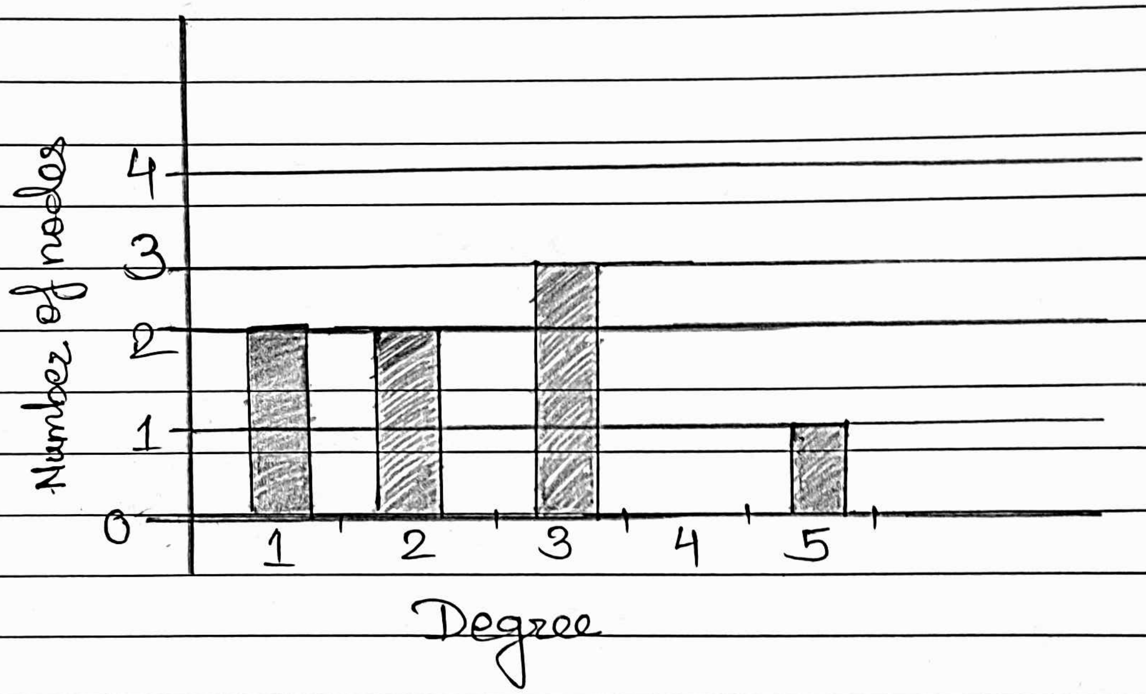
Step 3) Show a degree distribution in a bar graph. The x-axis has the degrees in ascending order, and the Y-axis indicate how many nodes have a given-degree.



m) What is the density of this graph?

Ans Formula to calculate density:

$\boxed{\text{number of edges} \div \text{number of possible edges}}$

for directed networks, the number of possible edges in a graph with n nodes is:

$$\boxed{n \times (n-1)}$$

In undirected networks, the number of possible edges is

$$\boxed{\frac{n \times (n-1)}{2}}$$

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∴ formulas to calculate density with n nodes and e edges is:

$$\boxed{\frac{e}{n \times (n-1)}} \rightarrow \text{directed network}$$

$$\boxed{\frac{e}{n \times (n-1)/2}} \rightarrow \text{undirected network}$$

The density for the given network

$$= \frac{e}{n \times (n-1)/2} = \frac{10}{8 \times (8-1)/2}$$

$$= \frac{10}{28}$$

$$= 0.357$$

n) Which node(s) have the highest degree? What is the degree?

Ans Node E have the highest degree with degree count of 5.

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o) Which node(s) have the lowest degree? What is the degree?

Ans There are 2 nodes having the lowest degree they are:
Node A and Node F

and Degree count of this node is 1

p) Which node has the highest degree centrality?

Ans Degree centrality of a node is simply its degree, the number of edges it has.

∴ Node E has the highest degree centrality with 5 edges to it.

q) Which node has the highest closeness centrality?

Ans Closeness centrality indicates how close a node is to all other nodes in the network. It is calculated as the average of the shortest path length from the node to every other node in the network

(1) For Node A

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Node	Shortest Path from A
B	1
C	2
D	2
E	3
F	4
G	3 (A→B→D→G)
H	4

The average of shortest path lengths is:

$= (1+2+2+3+4+3+4) \div 7$
$= 19 \div 7 = \underline{2.71}$

(2) For Node B

Node	Shortest Path from B
A	1
C	1
D	1
E	2
F	3
G	2
H	3

The average of shortest path lengths is:

$= (1+1+1+2+3+2+3) \div 7$
$= 13 \div 7 = \underline{1.85}$

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③ For Node C

Node	Shortest Path from C
A	2
B	1
D	2
E	1
F	2
G	2
H	2

The average of shortest path lengths is

$= (2+1+2+1+2+2+2) \div 7$

$= 12 \div 7$

$= 1.71$

④ For Node D

Node	Shortest Path from D
A	2
B	1
C	2
E	1
F	2
G	1
H	2

The average of shortest path lengths is

$= (2+1+2+1+2+1+2) \div 7 = 11 \div 7 = 1.57$

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(5) For Node E

Node	Shortest Path from E
A	3
B	2
C	1
D	1
F	1
G	1
H	1

The average of shortest path lengths is:

$= (3+2+1+1+1+1+1) \div 7$

$= 10 \div 7$

$= \underline{1.42}$

(6) For Node F

Node	Shortest Path from F
A	4
B	3
C	2
D	2
E	1
G	2
H	2

The average of shortest path lengths is

$= (4+3+2+2+1+2+2) \div 7 = 16 \div 7 = \underline{2.28}$

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7. For Node G

Node	Shortest Path from G
A	4
B	3
C	2
D	1
E	1
F	2
H	1

The average of shortest path lengths is:

$= (4+3+2+1+1+2+1) \div 7$
$= 14 \div 7$
$= \underline{2}$

8. For Node H

Node	Shortest Path from H
A	4
B	3
C	2
D	2
E	1
F	2
G	1

The average of Shortest path length is:
$= (4+3+2+2+1+2+1) \div 7 = 15 \div 7 = \underline{2.14}$

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In the case of closeness centrality, or average shortest path length, lower values indicate more central nodes.

∴ Comparing all Nodes average shortest path length

Node	Average shortest path lengths
A	2.71
B	1.85
C	1.71
D	1.57
E	1.42 ✓
F	2.28
G	2
H	2.14

∴ Node E has the highest closeness centrality.

r) What is the centralization (based on degree) of the graph?

Ans Step 1: Calculate centrality for every node in the network (use degree centrality)

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Node	Degree Centrality
A	1
B	3
C	2
D	3
E	5
F	1
G	3
H	2

Step 2: Measure the difference between the centrality of the most central node to each other node in the network, and add up these

Formula to compute centralization. It is equal to the sum of the differences divided by the maximum possible sum of differences.

$$\frac{\sum_{i=1}^{N} C(n^*) - C(n_i)}{\max \sum_{i=1}^{N} C(n^*) - C(n_i)}$$

The most central node is E

$\therefore$ The difference $E-A = 5-1 = 4$

$E-B = 5-3 = 2$

$E-C = 5-2 = 3$

E-D = 5-3 = 2
E-E = 5-5 = 0
E-F = 5-1 = 4
E-G = 5-3 = 2
E H = 5-2 = 3

Sum of differnce = 4+2+3+2+0+4+2+3
= 20

Step 3: Divide this by the maximum possible such sum for a network

$$\boxed{\therefore \max \sum_{i=1}^{N} c(n^*) - c(n_i) = (N-1)(N-2)}$$

$\therefore N = 8$

$= (8-1) \times (8-2)$

$= 7 \times 6$

$= 42$

$\therefore$ Centralization $= \frac{20}{42} = 0.476$

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s) What is the cohesion of the graph?

Ans

Connectivity, also known as cohesion, measures how those edges are distributed. Connectivity is a count of the minimum number of nodes that would have to be removed before the graph becomes disconnected; that is there is no longer a path from each node to every other node

∴ The connectivity is 1 because removing node E would disconnect the graph.

t) Compute betweenness centrality for a Node E.

Ans

Step 1: Select a pair of nodes and find all the shortest paths between those nodes.

Consider 10 pairs of nodes : AD, AE, AF, AG, AH, AC, CF, CH, CG, BE

	Shortest Path	Total	include E
① AD	A → B → D	1	0
② AE	A → B → D → E , A → B → C → E	2	0
③ AF	A → B → D → E → F , A → B → C → E → F	2	2
④ AG	A → B → D → G	1	0
⑤ AH	A → B → D → G → H, A → B → D → E → H, A → B → C → E → H	3	2
⑥ AC	A → B → C	1	0
⑦ CF	C → E → F	1	1

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8) CH – C→E→H (1) 1
9) CG – C→E→G (1) 1
10) BF – B→D→E→F, B→C→E→F (2) 2

Step2 : Compute the fraction of those shortest paths that include node M.
If there were 5 shortest paths between a pair of nodes, & 3 of them went through node M, then the fraction would be $3 \div 5 = 0.6$

$AD = 0 \div 1 = 0$
$AE = 0 \div 2 = 0$
$AF = 2 \div 2 = 1$
$AG = 0 \div 1 = 0$
$AH = 2 \div 3 = 0.66$
$AC = 0 \div 1 = 0$
$CF = 1 \div 1 = 1$
$CH = 1 \div 1 = 1$
$CG = 1 \div 1 = 1$
$BF = 2 \div 2 = 1$

Step3 : Add up the all fractions & this is the betweenness centrality for that node.

$= 0+0+1+0+0.66+0+1+1+1+1$
$= 5.66$